Numerical simulation of the scale effect on particle breakage in rockfill dams

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ABSTRACT
Rockfill dams present a challenge for engineers due to the many uncertainties revolving around the behavior of these geotechnical structures. A governing factor in the behavior of rockfill is the particle breakage due to change of moisture, as observed in laboratory and field conditions. Many constitutive models exist for soils but rockfill remains yet a relatively unexplored area and deserves more attention. The particle breakage phenomenon has been incorporated in a rockfill compressibility constitutive model by Alonso and Oldecop, where the suction inside the cracks of the rockfill is a state variable that controls this mechanism. A numerical study on a well instrumented rockfill dam is conducted using this constitutive model. The dam consists of rockfill shoulders and a central clay core, and has experienced considerable collapse settlement due to impoundment and rainfall. The instrumentation data provide an excellent opportunity to examine the state-of-the-art modeling techniques for settlement response of rockfill dams. The simulation of the stage construction and impoundment phase is carried out using the above model in Code_Bright, which is a fully coupled three-phase finite element program for unsaturated porous media. Simulation results are used to examine a recently proposed scaling law by Oldecop and Alonso for the compressibility of rockfill. The study concludes with comments on the accuracy of the scaling law and ideas for future developments.

RÉSUMÉ
Les barrages en enrochement présentent un défi pour les ingénieurs en raison des nombreuses incertitudes qui tournent autour le comportement de ces structures géotechniques. Un facteur déterminant dans le comportement des enrochements est la rupture des particules à cause du changement de l'humidité, comme a été observé dans des essaies de laboratoire et sur le terrain. Plusieurs modèles constitutifs existent pour les sols, mais les enrochements restent encore un domaine relativement inexplorée. Le phénomène de rupture des particules a été incorporé dans un modèle constitutif développé par Alonso et Oldecop, où la succion à l'intérieur des fissures de l'enrochement est une variable d'état qui contrôle ce mécanisme. Une étude numérique sur un barrage en enrochement bien instrumenté a été réalisée en utilisant ce modèle constitutif. Le barrage est constitué des épaulements rocheux et un noyau d'argile, et a subi un effondrement considérable à cause de la mise en eau et des précipitations. Les données récupérées de l'instrumentation fournissent une excellente occasion pour examiner le tassement des barrages avec ces techniques de modélisation de pointe. La simulation de l'étape de construction aussi comme de la phase de remplissage du barrage sont effectués à l'aide du modèle susmentionné avec Code_Bright, qui est un logiciel par éléments finis à trois phases entièrement couplées pour les milieux poreux insaturés. Les résultats des simulations sont utilisés pour examiner une loi d'échelle récemment proposée par Oldecop et Alonso pour la compressibilité des enrochements. L'étude conclut avec des commentaires sur l'exactitude de la loi d'échelle et des idées sur des travaux futurs.

1 INTRODUCTION

Rockfills in various engineering structures have demonstrated a peculiar behavior called collapse settlement which primarily depends on the amount of relative humidity (water content) and stress level. Studies indicate that there are two mechanisms that are involved in collapse behavior, namely particle re-arrangement and particle breakage.

Particle breakage which is of concern in this paper occurs (beyond a certain level of stress) due to unsaturated effects such as change in the humidity (water content) in the existing cracks in rock particles upon rainfall or inundation. Alonso and Oldecop (2001) have tackled this issue in the framework of continuum mechanics by introducing an elasto-plastic constitutive model using the concept of "compressibility" despite the fact that rockfills usually consist of very large discrete particles. The rockfill compressibility concept originates from the fact that the amount of particle breakage (hence, the amount of collapse settlement) depends on the size of the rock particles and larger particles create more compressible rockfills because of having higher void ratios on one hand, and weaker rocks with more inherent cracks on the other.

Quantification of the rockfill compressibility, however, is difficult because of the scale effect. The compressibility coefficient obtained from large scale oedometer tests in laboratory is different from the actual compressibility values of the rockfill because the real size of rock particles (which are sometimes even more than 1.5 m) cannot be examined in oedometer tests (Marsal 1873). This paper is part of a research project, trying to close this gap in knowledge by using computer simulation of the behavior of a well-instrumented rockfill dam and determining the “scale effect” that governs the discrepancy between the lab and field compressibilities. The approach adopted here is calibrating the numerical modeling results against the observed values of the rockfill collapse from the instrumentation data in the field. The rockfill dam considered for this study is Denis-Perron or SM-3 Dam spanning the Sainte-Marguerite river in eastern Quebec, Canada. Standing 171 m high and 378 m long, the dam is the primary component of Hydro-Quebec Sainte-Marguerite-3 Hydroelectric complex. The instrumentation data from the dam during the construction and impoundment provides this excellent opportunity to examine the effect of the size of the particles on the behaviour of the rockfill through numerical analysis. By employing the procedure proposed in this paper to find the appropriate scaling factor, the collapse behavior of rockfill is shown to be well captured using the elastoplastic constitutive model proposed by Alonso and Oldecop (2001).

2 ROCKFILL COMPRESSIBILITY MODEL

The model used in the numerical simulation is an elastoplastic constitutive model based on the main principles of the Barcelona Basic Model (BBM) (Alonso and Oldecop 1991). The model captures the volumetric deformation behavior of the rockfill that is based on a fracture propagation mechanism. This deformation mechanism is able to give a qualitative physical explanation of time-dependent strains and collapse strains of rockfill, and of their simultaneous dependence on stress and water action. Collapse strains in dams usually occur during reservoir impoundment. This supports the idea that the presence of water within the particle cracks erodes the bonds, causing them to propagate and eventually break the particle. This mechanism is called particle breakage (Nobari and Duncan 1972). The main factors influencing the particle breakage phenomenon are 1) particle size 2) rock strength 3) grain size uniformity 4) compaction and 5) initial moisture content (Oldecop and Alonso 2013). The particle breakage in the model is related to the stress level in the material and linked to the relative humidity of the particles, i.e., suction stresses in the particle’s cracks. The major parameter controlling this phenomenon is the suction variable, denoted by *s,* which in the case of rockfill refers to the total suction instead of matric suction as in unsaturated soils. The suction is a result of the change in the relative humidity with time, causing the breakage of the stressed particles and is interpreted as time dependent volume change (Clements 1981).

Particle breakage is found to be related to the size of the particles - the larger the particle size, the weaker it becomes due to an increased amount of flaws and cracks. The volume change of the material with larger particle sizes, for the same stress level, is greater than that of material with smaller particle sizes. Therefore, the size of the particles is directly connected with the volumetric behavior of rockfill and thus to the materials compressibility.

The model considers a linear relationship for the stress-strain response for both instantaneous and time dependent deformation of the material based on experimental data (Oldecop and Alonso 2001). The model is based on isotropic compressibility, elastic behavior, yield stress function with a volumetric hardening law, critical state and has an extension for triaxial stress conditions with a yield and plastic potential functions. For the purposes of this paper, only the components of the constitutive model that include compressibility parameters are discussed due to the established connection of compressibility with scale effects.

The volumetric compressibility is assumed to be governed by two components. Under a threshold stress value, p_y, only the first mechanism occurs. It is called particle re-arrangement and involves slip and rotation of the particles in relation to their neighbours. The second mechanism is active beyond that threshold stress value and controls particle breakage. Isotropic compressibility is described by Equations 1.1 and 1.2:

$$p \leq p_y \quad d\varepsilon_v = \lambda^i dp \qquad [1.1]$$

$$p \geq p_y \quad d\varepsilon_v = [\lambda^i + \lambda^d(s)]dp \qquad [1.2]$$

where $d\varepsilon_v$ is the incremental volumetric strain, p is the total mean stress and λ^i is a compressibility parameter, where the superscript i stands for instantaneous deformation. The compressibility parameter $\lambda^d(s)$ represents the particle breakage mechanism, which is dependent on the total suction and captures some of the macroscopic phenomena observed in laboratory testing. The compressibility $\lambda^d(s)$ increases when the rockfill is wetted (decrease of suction) and thus starts to collapse at a constant stress. The superscript d shows the delayed nature of the particle breakage mechanism. Equation 2 appropriately captures this behavior:

$$\lambda^d(s) = \lambda_0^d - \alpha_s \ln[(s + p_{atm})/p_{atm}] \qquad [2]$$

where λ_0^d is the maximum clastic compressibility at saturated conditions and α_s is a material parameter.

The volumetric elastic strain of rockfill is dependent on the two compressibility coefficients κ and κ_s and the Poisson ratio, υ. The swelling index κ_s is negligible in the examined material and thus ignored here. κ is defined as follows:

$$d\varepsilon_v^e = \kappa dp = 3dp(1 - 2\upsilon)/E \qquad [3]$$

The yield stress, $p_0(s)$, is a function of the suction and is defined in the following way:

For $p_0^* \leq p_y$: $p_0(s) = p_0^*$ [4.1]

For $p_0^* > p_y$:

$$p_0(s) = p_y + \left[(\lambda^i - \kappa)(p_0^* - p_y)\right]/\left[\lambda^i + \lambda^d(s) - \kappa\right] \qquad [4.2]$$

where p_0^* is the yield stress for the very dry rockfill material. The model follows a volumetric hardening law that describes the evolution of p_0^*:

$$dp_0^* = d\varepsilon_v^p/(\lambda^i - \kappa) \quad [5]$$

where the volumetric plastic strain $d\varepsilon_v^p = d\varepsilon_v - d\varepsilon_v^e$.

The next section of the paper will address the compressibility parameter governing the particle breakage mechanism and the effect of particle size on it.

3 SIZE EFFECT IN ROCKFILL

The objective of the paper is to address the effect of particle size on the compressibility parameters through a numerical simulation. First, it has to be established based on previous research the connection between particle size and the compressibility of rockfill and other potential factors that are involved.

Tests show that the external stress capable of breaking particles, $(\sigma_{ext})_f$, depends on the particle size and is defined like

$$(\sigma_{ext})_f \propto d^{-\alpha} \quad [6]$$

where d is the average particle size and α varies between 0.3 and 0.5 for the tested materials. For more information on the derivation, refer to Oldecop and Alonso (2013). An explanation for the phenomenon are the more defects, cracks and micro-cracks in a bigger particle than a smaller one, which provides more stress concentration zones and thus weakens the particle. Therefore the bigger the particle, the more particle breakage occurs. Large oedometer tests are impractical and expensive and the maximum grain size that can be tested is 150-200 mm (Alonso et al. 2012). Most dams have a maximum particle size of 0.5-1 m, which is impossible to test in a laboratory. Grain size distributions may be scaled down in an attempt to preserve the behavior of the sample dimensions (Marachi et al. 1969), but the scale effects still have to be addressed.

Assuming assemblies of uniform spherical particles and linear stress-strain relationship, a scaling law similar to Equation 6 is proposed by Oldecop and Alonso (2013) for the delayed compressibility parameter:

$$\lambda \propto d^{\alpha} \quad [7]$$

$$\lambda^d = \lambda_0^d (d/d_0)^{\alpha} \quad [8]$$

Equation 8 follows directly from Equation 7, where d is the maximum particle size of the prototype (field) material; d_0 is the maximum particle size of the model (laboratory) material; λ^d is the compressibility parameter of the prototype and λ_0^d is the compressibility parameter of the model. The exponent α is a function of the density of the aggregate and the type of rock and governs the severity of scaling. In Oldecop and Alonso (2013) a limestone material was presented, which has α values varying between 0.33 and 0.5 for dense and loose aggregates, respectively. Based on Equation 8, a plot on Figure 1 is presented to show the variation of λ^d with different α values, where $SF = d/d_0$ is a scaling factor, relating prototype dimensions to model dimensions.

Scaling up of λ_0^d is necessary because it governs the particle breakage mechanism, which as discussed before depends on the particle size. The parameter λ^i on the other hand, governs the particle re-arrangement mechanism, which is not reported to be dependent on the particle size.

Figure 2 shows the scaling law from Equation 8 applied to laboratory data. The laboratory data is related to the compressibility of two sets of samples, both with uniform particle sizes (40-30 mm, 30-20 mm; 25-20 mm; 20-10 mm) but different densities (one loose with *e*=0.947 and one dense with *e*=0.5). The scaling law of Equation 8 is used by Alonso and Oldecop (2013) to scale down

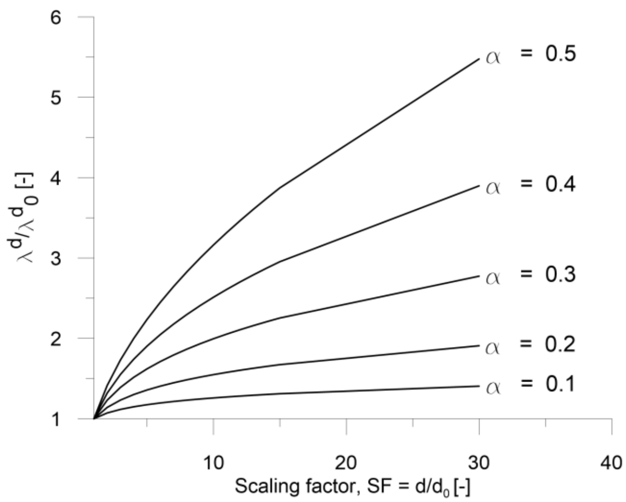

Figure 1. Variation of compressibility parameter λ^d for different α values

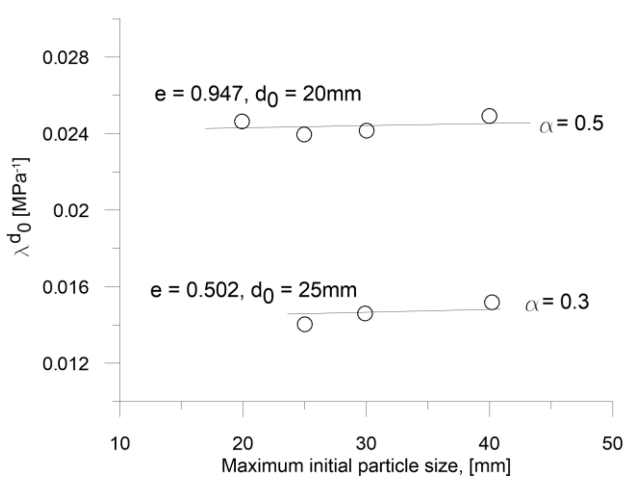

Figure 2. Corrected compressibility of limestone material to account for scale effect (modified from Oldecop and Alonso 2013)

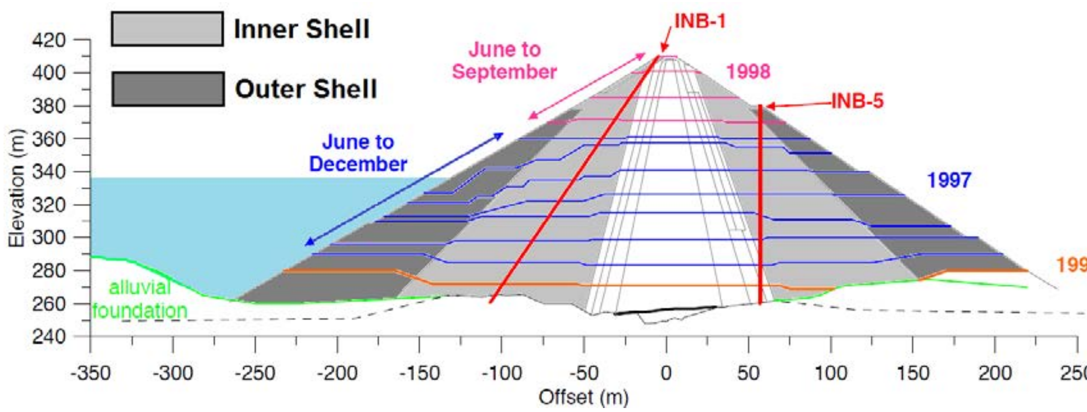

Figure 3. Cross section of SM-3 dam illustrating the construction sequence and locations of the inclinometers INB1 in upstream and INB5 in downstream (Modified from Errecalde 2012)

the λ^d of each sample to the corresponding λ_0^d of the sample with the smallest particle size. In this process they found that "taking d_0 as the minimum particle diameter tested, the size effect disappears, provided $\alpha = 0.5$ for the loose gravel and $\alpha = 0.33$ for the dense aggregate". The conclusion from this process was that "the α coefficient and, therefore, the intensity of scale effects, depend on aggregate density". This dependence will be further examined in the present paper using data available from an instrumented rockfill dam.

4 NUMERICAL MODEL DESCRIPTION

The Denis-Perron dam has a central till core, filters and transitions that are rested on concrete. The shoulders are consisted of rockfill, with particle sizes that can reach up to 1.8 m. A cross-section of the Denis-Perron dam is shown in Figure 3, illustrating the dimensions, general layering, construction timing, and locations of two inclinometers. The inner and outer shells of the dam are made of high-quality rockfill material, composed of anorthosite and biotite rocks. The material is compacted and the void ratio is estimated to be around 0.37. The void ratio is calculated based on a reported saturated unit weight, $\gamma_{sat} = 22$ kN/m^3, and $G_s = 2.7$ and is assumed to be the same value for both rockfill materials (Hydro-Quebec 1996). The maximum particle diameter, D_{max}, values for the inner and outer shells of the dam are shown in Table 1.

Table 1. Maximum particle size for different rockfill zones of the dam

Rockfill Zones	D_{max} [mm]
Inner shell	900
Outer shell	1800

Oedometer tests were performed on dry and saturated material with a reported scaled grain size distribution, where the coefficient of uniformity, C_u, of the oedometer and the field material was preserved (Errecalde 2012). The oedometer device had a diameter of 300 mm; the maximum particle size for the tested material is estimated to be 1/5th of that, i.e. about 60 mm. Based on the saturated and dry tests, values for the compressibility parameters λ_0^d, $\lambda_i - \kappa$ were determined as 0.009 and 0.01 MPa^{-1}, respectively.

The numerical simulation is set up in a finite element platform called Code_Bright, developed at the Polytechnic University of Catalonia (Code Bright, 2015). This is a three dimensional FEM software for coupled Thermo-Hydro-Mechanical analysis in geological media. The model of the dam was built layer by layer in accordance with real life construction and impoundment stages. The whole simulation captures the construction and full impoundment of the dam reservoir, lasting for 5 years. A detailed description of the finite element model, including mesh, initial and boundary conditions and constitutive model calibrations will be accessible in Kolev (2016), and are not presented in the present short paper.

5 SIMULATION RESULTS AND DISCUSSION

To evaluate the performance of the numerical simulation, several results are compared with the corresponding ones from the instrumentation data such as inclinometers, piezometers, pressure cells, and displacement gauges. For the purposes of this paper, selected settlement results are compared with the two inclinometers shown on Figure 3, i.e., INB1 and INB5. INB1 is an inclined inclinometer located in the upstream side of the dam, where impoundment causes larger settlements. INB5 is a vertical inclinometer located in the downstream section of the dam, where the rockfill response is not affected by the water impoundment due to the action of the core.

The simulation results are compared to the field measurements at three different times during construction and impoundment: i) at construction up to elevation 325 m and no impoundment; ii) at construction up to elevation 360 m and reservoir level risen to 292 m; iii) at completed dam construction and reservoir level at 334 m.

Comparison of the simulation results to the settlement measurements at INB1 and INB5 has been done initially

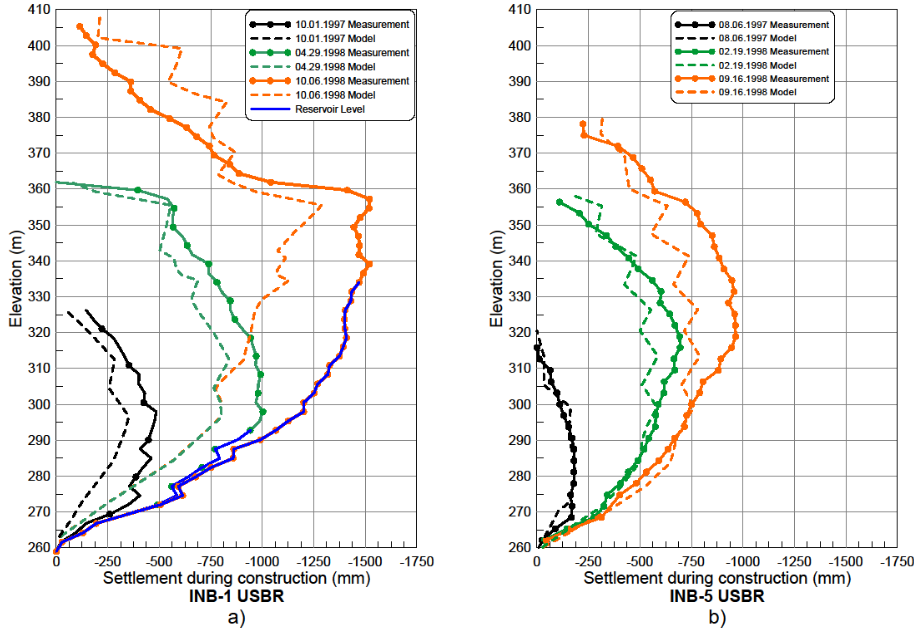

Figure 4. Calculated and measured vertical displacements at different elevations (a) INB1 upstream and (b) INB5 downstream during three different stages of construction and impoundment. Compressibility parameter λ_0^d unscaled

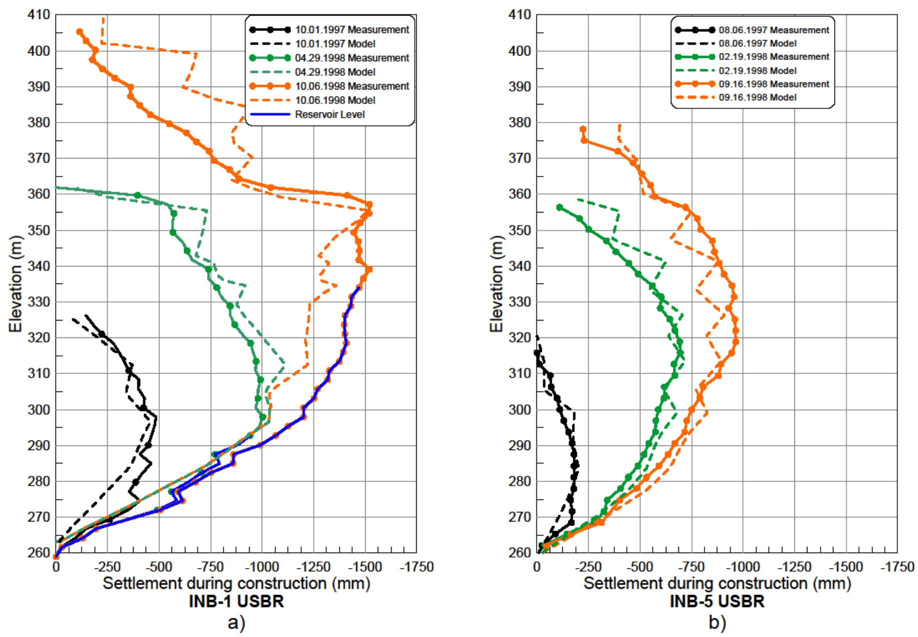

Figure 5. Calculated and measured vertical displacements at different elevations (a) INB1 upstream and (b) INB5 downstream during three different stages of construction and impoundment. Compressibility parameter λ_0^d scaled

Table 2. Compressibility parameters [MPa^{-1}] for three existing rockfill structures

	Beliche Dam*		Lechago Dam**		Denis-Perron Dam	
	Inner Shell	Outer Shell	Inner Shell	Outer Shell	Inner Shell	Outer Shell
$\lambda_i - \kappa$	0.025	0.010	0.03	0.03	0.010	0.010
λ_0^d unscaled	0.028	0.010	0.03	0.03	0.009	0.009
λ_0^d scaled	0.042	0.015	No scaling	No scaling	0.015	0.017

* Alonso and Oldecop (2005)
**Alonso and Olivella (2011)

with using the $\lambda_0^d = 0.009$ directly calibrated based on the oedometer test data and no scaling applied. The results of this simulation are presented in Figures 4(a) and 4(b) for the INB1 and INB5, respectively. These results show under-prediction of the settlements recorded at INB1 in the zones where the rockfill has been exposed to the water action but good predictions otherwise for both inclinometers. Judging by the results from this figure, the compressibility parameter λ_0^d responsible for the wet behaviour of rockfill has to be increased, or in other words scaled up from the one calibrated based on the oedometer test. This is supported also by the background discussed in Section 3.

Both inclinometers are predominantly within the inner shells of the dam. Therefore, the effect of the outer shell settlements cannot be captured by the inclinometers. Hence, the more representative value λ_0^d for the inner shell is estimated based on comparing the settlement from the numerical simulations and the data of INB1 and INB5. The best match of the settlements is achieved with scaling the λ_0^d of the inner shell by a factor of 1.65 from the value of 0.009 obtained in the lab test, resulting in $\lambda_0^d = 0.015$. The simulation results using this revised value of λ_0^d are presented in Figures 5(a) and 5(b). The new results illustrate that the scaled λ_0^d provides an improved approximation of the field settlements with better agreement to the field data, hence better capturing the compressibility of large wet particles.

Similar approach was undertaken in the past for Beliche Dam, where the compressibility was scaled up by a factor of 1.5 for both inner and outer shells (Alonso et al., 2005). Another example of a rockfill dam is Lechago. In the case of Lechago, however, the in-situ material was tested in laboratory by removing some of the large particles and no scaling was applied in the related analyses (Alonso et al. 2012). A summary of the compressibility parameters for these three rockfill dams is presented in Table 2. With reference to Figure 1, the parameter α for the rockfill material of the SM-3 dam can be calculated when the scaled λ_0^d (presented above) and field to laboratory SF are known for the material. For the inner and outer shells, the values of SF have to be calculated based on their maximum particle sizes. Assuming the maximum particle size tested in the oedometer is about 60 mm, as discussed in Section 4, the $SF = d/d_0$ can be calculated, where d_0 is the maximum particle size tested and d is the D_{max} of the material from Table 1. For the inner shell, $SF = d/d_0 = 900/60 = 15$, and for the outer shell $SF = d/d_0 = 1800/60 = 30$. Based on this information and given the obtained factor of 1.65 for scaling of the compressibility parameter of the inner shell, using Figure 1 the parameter α is estimated as 0.19 for the inner shell. As mentioned earlier, the outer shell scaling cannot be calibrated using the results from INB1 and INB5. Hence, the outer shell rockfill is assumed to follow a similar scaling intensity parameter $\alpha = 0.19$ as the inner shell, but a different scaling factor $SF = 30$. Using those parameters and Figure 1, the λ_0^d of the outer shell is estimated to increase by a factor of 1.9 to 0.015 as shown in Table 2. The simulation results presented in Figure 5 included also this scaled compressibility parameter for the outer shell, but presumably the scaling of the outer shell has minimal effect on the settlement results at INB1 and INB5 because to their placement being almost entirely within the inner shell. The λ_0^d of the outer shell needs to be studied more carefully based on additional instrumentation data within this zone to get the real scaling and thus a more reliable value for α, but this will be addressed further in Kolev (2016).

Earlier it was shown in Figure 2 that α seems to depend on the material density (or void ratio). The present study suggest $\alpha = 0.19$ that for the case of Denis-Perron dam with an approximate void ratio of 0.37 for the inner shell. This information when combined with the already reported pairs of void ratio and α from Figure 2, can lead to the apparent linear relation between these two quantities as shown in Figure 6. Of course the limited data points available in this figure are not sufficient to draw a definitive conclusion about the relation between void ratio and α , but this figure opens the door for additional future investigations about the effect of density (void ratio) on the intensity of scaling through the exponent α. More "points" should can be added to this figure from other laboratory experiments (as reported in Oldecop and Alonso 2013), or from the numerical analysis of instrumented large rockfill structures like Beliche dam or Denis-Perron dam (as done in the present study).

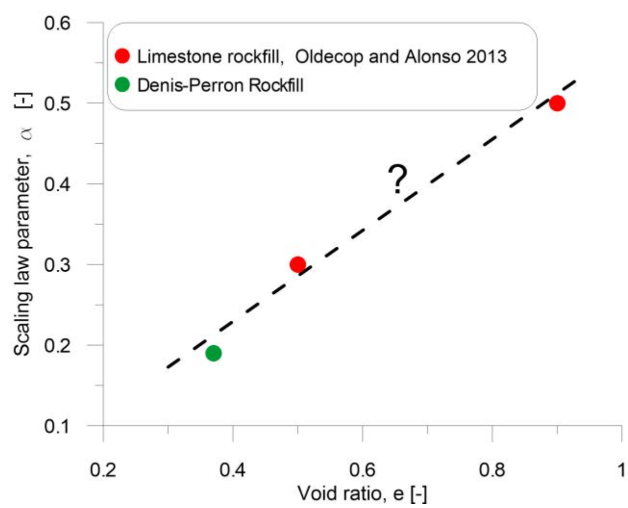

Figure 6. Variability of parameter α based on void ratio for laboratory data (Oldecop and Alonso 2013) and simulation results

6 CONCLUSIONS

The sensitivity of rockfill to water action is well known from the previous experiments and existing structures. A constitutive model developed by Alonso and Oldecop (2001) was used. One of the key parameters that govern the mechanical behavior of the rockfill model is the compressibility index, λ_0^d, for saturated conditions. One would need to account for the scale effects of rockfill particles on the compressibility parameter. A numerical simulation of Denis-Perron dam was used to gain insight into the scaling of the compressibility parameter applicable to the rockfill material of this dam. The analysis was carried out initially using the unscaled compressibility parameter, and then was repeated with different scaled λ_0^d values until the simulation results matched the field measurements for settlements. The calibrated λ_0^d values that account for the scale effects, were then compared to the corresponding laboratory value. The comparison was done based on an idealized scaling law of uniform particle packing with linear stress-strain relation as proposed by Oldecop and Alonso (2013). The law is governed by an exponent α that was earlier reported to depend on the density (or void ratio). This paper presented a successful simulation of the response of Denis-Perron dam during construction and first impoundment, in comparison with the instrumentation data at two different inclinometers. It also provided additional information for exploring the expected relation between the void ratio and the exponent α for rockfill materials. Further investigation should be done to better populate Figure 6 and get a better understanding of this particular scaling mechanism.

ACKNOWLEDGEMENTS

The materials presented in this paper are part of a comprehensive study at the University of British Columbia on numerical modeling of response of rockfill dams, with partial support provided by the Natural Sciences & Engineering Research Council of Canada (NSERC). Daniel Varret, Eric Peloquin, and Annick Bigras of Hydro-Quebec provided technical insight to the instrumentation data in the extended version of this work that will be presented in a future publication.

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